Period 1, April 29, 2024

$6xy = 2 + y^3.$

$$6\frac{dx}{dT}\cdot y + 6x\cdot\frac{dy}{dT} = 0 + 3y^2\frac{dy}{dT}$$

$$6\frac{dx}{dT}(-2) + 6\left(\frac{1}{2}\right)\frac{dy}{dT} = 3(-2)^2\frac{dy}{dT}$$

$$6\cdot\frac{2}{3}(-2) + 3\frac{dy}{dT} = 12\frac{dy}{dT}$$

$$-3\frac{dy}{dT} \qquad -3\frac{dy}{dT}$$

$$-8 = 9\frac{dy}{dT}$$

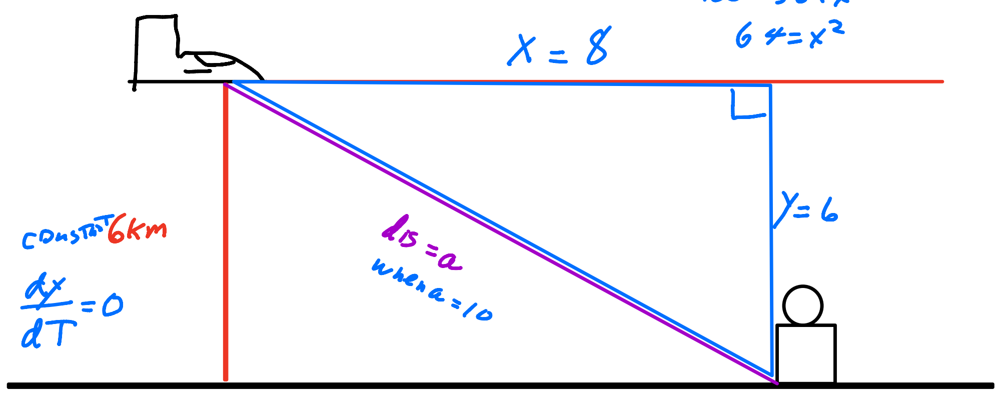
$$\frac{-8}{9} = \frac{dy}{dT}$$

$\frac{-8}{9}$ units per second

$10^2 = 6^2 + x^2$

$100 = 36 + x^2$

$64 = x^2$



$$\frac{da}{dT} = -400\ \text{km/h}$$

$$\frac{dx}{dT} = ?$$

$$x^2 + y^2 = a^2$$

$$2x\frac{dx}{dT} + 2y\frac{dy}{dT} = 2a\frac{da}{dT}$$

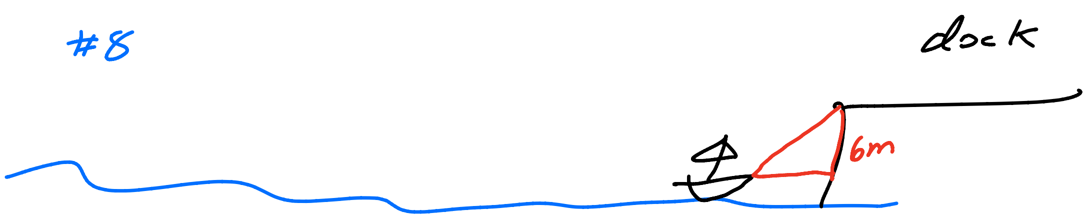
$$2(x)\frac{dx}{dT} + 2(6)(0) = 2\cdot 10\text{km}\left(-400\ \text{km/h}\right)$$

$$2x\frac{dx}{dT} = -8000\ \text{km}^2/\text{h}$$

$$\frac{2\cdot 8\text{km}\cdot\frac{dx}{dT}}{16\ \text{km}} = \frac{-8000\ \text{km}^{\cancel{2}}/\text{h}}{16\ \cancel{\text{km}}}$$

$$\frac{dx}{dT} = -500\ \text{km/h}$$

#8



~~20~~, ~~22~~, ~~23~~, ~~25~~, ~~33~~, 45, 44, 38, ~~30~~, ~~40~~
43, ~~41~~, ~~26~~, ~~15~~, 6, ~~24~~, ~~34~~

6. $$\lim_{x\to 0}\frac{x-\sin x}{x^2}=\frac{0-\sin 0}{0^2}=\frac{0-0}{0}=\emptyset$$

$$\lim_{x\to 0}\frac{1-(\cos x)}{2x}=\frac{1-\cos 0}{2(0)}=\frac{1-1}{0}=\frac{0}{0}=\emptyset$$

$$\lim_{x\to 0}\frac{0-(-\sin x)}{2}=\frac{0}{2}=0$$

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11. $$F(x)=\sqrt{\operatorname{Tan}(5x)}=(\operatorname{Tan}5x)^{\frac{1}{2}}$$

$F'(\frac{\pi}{4})$ $$F'(x)=\frac{1}{2}(\operatorname{Tan}5x)^{-\frac{1}{2}}\cdot\sec^2 5x\cdot 5$$

$\operatorname{Tan}\frac{\pi}{4}=1$ $\sec\frac{\pi}{4}=\sqrt{2}$ $$\frac{5\sec^2 5x}{2\sqrt{\operatorname{Tan}5x}}$$

$$y=\sqrt{\operatorname{Tan}5x}=(\operatorname{Tan}5x)^{\frac{1}{2}}\Rightarrow y=(\operatorname{Tan}u)^{\frac{1}{2}}\Rightarrow y=L^{\frac{1}{2}}$$

$u=5x$ $\quad L=\operatorname{Tan}u$ $\quad \frac{dy}{dL}=\frac{1}{2}L^{\frac{1}{2}-1}=\frac{1}{2}L^{-\frac{1}{2}}$

$\frac{du}{dx}=5$ $\quad \frac{dL}{du}=\sec^2 u$

$$\frac{\cancel{du}}{dx}\cdot\frac{\cancel{dL}}{\cancel{du}}\cdot\frac{dy}{\cancel{dL}}=5\cdot\sec^2 u\cdot\frac{1}{2\sqrt{L}}=\frac{5\sec^2 5x}{2\sqrt{\operatorname{Tan}5x}}$$

$\operatorname{Tan}\frac{5\pi}{4}=1$

$\sec\frac{5\pi}{4}=-\sqrt{2}$

$$\frac{5\cdot(-\sqrt{2})^2}{2\sqrt{1}}=\frac{5\cdot 2}{2}=5$$

15.

$$\lim_{x\to\infty} \frac{\ln(x+1)}{\log_2 x} = \frac{\ln\infty}{\log_2\infty} = \frac{\infty}{\infty} = \emptyset$$

$$\lim_{x\to\infty} \frac{\frac{1}{x+1}}{\frac{1}{x\ln 2}}$$

$\log_2 x = y$

$2^y = x$

$\ln 2^y = \ln x$

$\frac{y \cdot \cancel{\ln 2}}{\ln 2} = \frac{\ln x}{\ln 2}$

$y = \frac{1}{\ln 2} \cdot \ln x$

$\frac{dy}{dx} = \frac{1}{\ln 2} \cdot \frac{1}{x}$

$$\lim_{x\to\infty} \frac{1}{x+1} \cdot \frac{x\ln 2}{1}$$

$$\approx \frac{1}{\cancel{x}} \cdot \frac{\cancel{x}\ln 2}{1} = \ln 2$$

20.

$$\int_0^{\pi/2} \sin 2x \, e^{\sin^2 x} dx = \int \sin 2x \cdot e^u dx$$

$$\int \cancel{2\sin x\cos x} \cdot e^u \cdot \frac{du}{\cancel{2\sin x\cos x}}$$

$du = 2\cos x \sin x dx$

$u = \sin^2 x$

$\frac{du}{2\cos x\sin x} = dx$

$u = L^2 \Rightarrow \frac{du}{dL} = 2L$

$L = \sin x$

$\frac{dL}{dx} = \cos x$

$\frac{\cancel{dL}}{dx} \cdot \frac{du}{\cancel{dL}} = \frac{du}{dx}$

$\cos x \cdot 2L = 2\cos x \cdot \sin x$

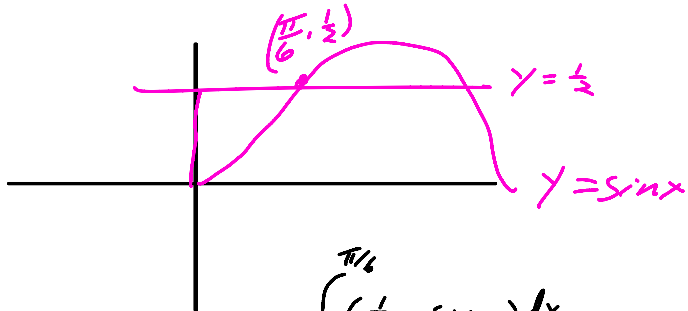
$e^u + C$

$e^{\sin^2 x} + C \Big]_0^{\pi/2}$

$e^{1^2} + \cancel{C} - e^0 - \cancel{C}$

$e - 1$

22.



$$\int_0^{\pi/6} \left(\tfrac{1}{2} - \sin x\right) dx$$

$$\frac{1}{2}x - (-\cos x) + C \Big|_0^{\frac{\pi}{6}}$$

$$\frac{1}{2} \cdot \frac{\pi}{6} + \cos\frac{\pi}{6} = \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$$

$$-\left(\tfrac{1}{2}(0) + \cos 0\right)$$

$$-1$$

$$\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$$

23

$$F(x) = x^4 + 4x^3$$

$$m = F'(x) = 4x^3 + 12x^2$$

$$0 = 4x^2(x+3)$$

$x = 0$

$x = -3$

decreasing $m = -$

$(-\infty, -3)$

$$\begin{array}{ccccccc} - & - & + & + & + & + & + \\ -4 & -3 & -2 & -1 & 0 & 1 & 2 \end{array}$$

$F'(-4) = 4(-4)^2(-4+3) = + \cdot - = -$

$F'(-1) = 4(-1)^2(-1+3) = + \cdot +$

$F'(1) = 4(1)^2(1+3) = + \cdot +$

24.

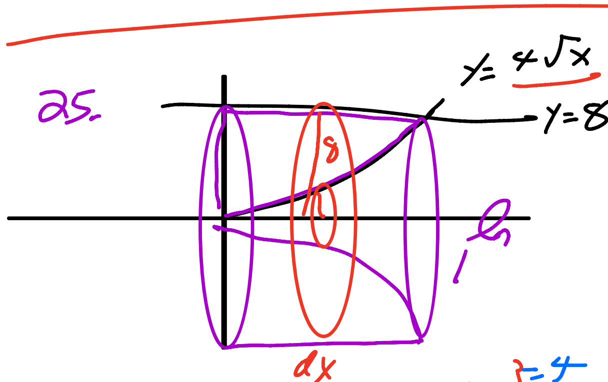
$$\lim_{x\to 0} \frac{\tan 3x + 3x}{\sin 5x} = \frac{0+0}{0} = \frac{0}{0} = \not{0}$$

$$\frac{3 \cdot \sec^2 3x + 3}{5 \cdot \cos 5x} = \frac{3(1)^2+3}{5 \cdot 1} = \frac{6}{5}$$

$\sec 0 = 1$

$\cos 0 = 1$

25.



$y = 4\sqrt{x}$

$y = 8$

$R = 8$

$r = 4\sqrt{x}$

$$\frac{\cancel{4}\sqrt{x}}{\cancel{4}} = \frac{8}{4}$$

$\sqrt{x} = 2$

$x = 4$

$$\int_0^{?=4} \pi\left(8^2 - (4\sqrt{x})^2\right)dx$$

$$\pi \int_0^4 (64 - 16x)\,dx$$

$$\pi(64x - 8x^2)\Big|_0^4$$

$$\pi\left[(64\cdot 4 - 8\cdot 16) - (64\cdot 0 - 8(0)^2)\right]$$

$$\pi(256 - 128) = 128\pi$$

26

$S(T) = 2T^3 - 12T^2 + 16T + 2$

$V(T) = S'(T) = 6T^2 - 24T + 16$

Max $V(T)$

$V'(T) = 0 = 12T - 24 = 12(T-2)$

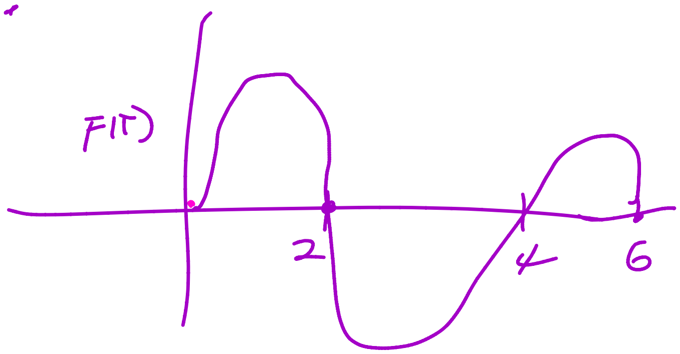
$T = 2$

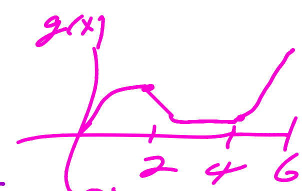
$[0, 5] = T$

T	$V(T)$	
0	16	$= 6(0)^2 - 24(0) + 16$
2	-8	$6(2)^2 - 24(2) + 16$
5	46	$6(5)^2 - 24(5) + 16$
		$150 - 120 + 16$

$F(T) = S'(x)$ $\quad g(x) = \int_0^x F(T)\,dT$

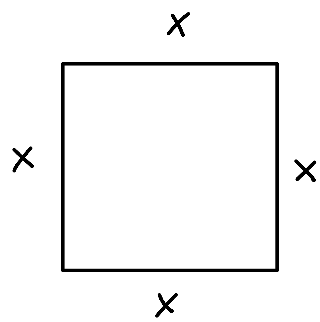
30.





B

33.



$\frac{dx}{dT} = 0.4 \text{ cm/s}$

$P = 4x$

$\frac{dP}{dT} = 4\frac{dx}{dT}$

$\frac{dP}{dT} = 4\frac{dx}{dT} \Leftarrow P = 4x \qquad \frac{P}{4} = x$

$A = x^2$

$A = \left(\frac{P}{4}\right)^2 = \frac{P^2}{16}$

$\frac{dA}{dT} = \frac{2P}{16} \cdot \frac{dP}{dT}$

$\frac{dP}{dT} = 4(0.4)$

$\frac{dP}{dT} = 1.6 \text{ cm/s}$

$\frac{1}{8}P \cdot 1.6$

B

$.2P$

34.

$F(x) = 3^x$

$\frac{dy}{dx} = 1$

$y = 3^x$

$\ln y = \ln 3^x$

$\ln y = x \cdot \ln 3$

$\cancel{y} \cdot \frac{1}{y}\frac{dy}{dx} = \ln 3 \cancel{y}$

$\frac{dy}{dx} = (\ln 3) 3^x$

$\frac{1}{\ln 3} = \frac{\cancel{\ln 3} \cdot 3^x}{\cancel{\ln 3}}$

$.910239 = 3^x \Rightarrow \ln .910239 = \ln 3^x$

$\frac{-0.094048}{\ln 3} = \frac{x \cancel{\ln 3}}{\cancel{\ln 3}}$

-0.085606

38. $F(x) = x^4 - 3x^2 + 5$

$F'(x) = 4x^3 - 6x$

$F''(x) = 12x^2 - 6$

$0 = 12x^2 - 6 = 6(2x^2 - 1)$

$x^2 = \frac{1}{2}$

$x = \pm \frac{\sqrt{2}}{2} = \pm\sqrt{\frac{1}{2}}$

concave up

$F''(x) = +$

$(-\infty, -\sqrt{\frac{1}{2}}) \cup (\sqrt{\frac{1}{2}}, \infty)$

+ + + + − − − − − + + +

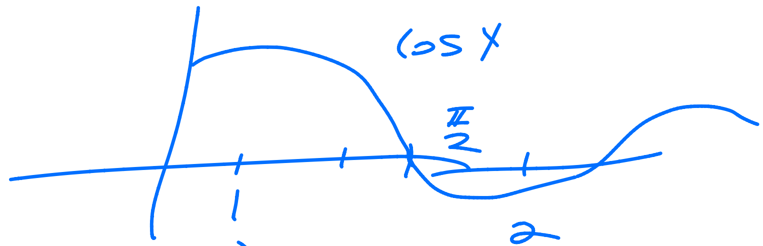
-1 $\quad -\sqrt{\frac{1}{2}}$ $\quad 0$ $\quad \sqrt{\frac{1}{2}}$ $\quad 1$

$F''(0) = -6$

$F''(1) = 6$

$F''(-1) = 6$

43.



$$\int_1^2 \cos x\,dx =$$

$$\int_1^2 |\cos x|\,dx = \int_1^{\frac{\pi}{2}} \cos x\,dx - \int_{\frac{\pi}{2}}^2 \cos x\,dx$$

$$\sin\frac{\pi}{2} - \sin(1) - \sin 2 + \sin\frac{\pi}{2} =$$

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44. $F(x) = \int \cot x\,dx \qquad 0 < x < \pi$

if $F(\frac{\pi}{6}) = 1$

$1 + \int_{\frac{\pi}{6}}^{1} \cot x\,dx$ Then $F(1)$

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45. $\dfrac{dy}{dx} = 2xy - y \qquad y(0)=4$

$\dfrac{dx}{y} \cdot \dfrac{dy}{\cancel{dx}} = \dfrac{y(2x-1) \cdot dx}{y} \qquad F(0)=4$

$\displaystyle\int \frac{1}{y}\,dy = \int (2x-1)\,dx$

$\ln y = x^2 - x + c$

$e^{x^2-x+c} = y \qquad e^{x^2-x} \cdot e^{c} = 4e^{x^2-x}$

$e^{0+0+c} = 4$

$e^{c} = 4$

D